

## Zavod za komunikacijske i svemirske tehnologije

Laboratorij za primjenjenu optiku

Unknown source analysis

Jakov Tutavac

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## Introduction

This document presents an analysis of the error in the spectral unmixing method, in the case when an unknown spectrum is present in the light mixture and degrades the results. This is relevant for the problem risk: spectra not in reference, for which the summary is given as follows.

The spectral unmixing algorithm uses reference spectra acquired on either the spectrometer or acquired using the light-pollution module in method of self-calibration. Measuring mixtures with artificial light sources not present in the calibration library will increase the error.

Two questions, which are answered at the conclusion, are given shortly. Given a list of light sources and the shape of their measured spectra, as acquired by a spectrometer.

- a) Which light source present in the mixture but missing from the spectral reference database, induces the largest root mean-square error?
- b) How many light sources present in the mixture can be missing from the spectral reference database, with the root mean-square error being less than 10 %?

The list of light sources available is:

- 1. High-pressure sodium (HPS)
- 2. High-pressure mercury (HPM)
- 3. Metal-Halide, CCT 3000 K (MH 3k)
- 4. Metal-Halide, CCT 4000 K (MH 4k)
- 5. White light-emitting diode, CCT 2700 K (LED 2k7)
- 6. White light-emitting diode, CCT 3000 K (LED 3k)
- 7. White light-emitting diode, CCT 4000 K (LED 4k)

- 8. White light-emitting diode, CCT 6400 K (LED 6k4)
- 9. Halogen light source (HAL)
- 10. Compact fluorescent lamp (CFL)

## Analysis

The shape of the spectra of each light source is known and given. We normalize the spectra by dividing each spectrum with the sum of its intensities, such that the integral of the spectrum is equal to one. In other words, each spectrum is normalized such that the total power over all wavelengths is equal to one.

Eleven Monte Carlo experiments are performed, and for each the removed light source denoted by number k is changed from k = (1, ..., 10). The spectral reference data bank, for the k-th experiment, contains all light sources, expect the k-th removed light source. For the last eleventh experiment, none of the light sources are removed and the spectral reference data bank contains all light sources.

Each experiment has 10 000 iterations, and each iteration consists of the following steps.

First, the fractions of each light source present in the mixture are randomly chosen. The sum of the fractions is equal to one.

Second, a mixture of light spectrum is synthesized by multiplying each spectrum from the list with a fraction and summing them together. Note, that the total power over all wavelengths of the synthesized mixture is equal to one.

Third, a non-negative sum constrained to one least-squares method is employed as the spectral unmixing procedure. This procedure has two inputs, the reference spectral data bank and the synthesized mixture, and one output: the vector containing estimated fractions.

Fourth (final step), the root-mean square error (RMSE) value between the randomly chosen fractions,  $\mathbf{a}$ , and the estimated fractions,  $\mathbf{x}$ , as given by the spectral unmixing method, is computed. Note, the first vector,  $\mathbf{a}$ , has 10 elements (number of light sources present in the mixture) while the second,  $\mathbf{x}$ , has 9 elements (number of light sources present in the spectral reference data bank). A new vector,  $\mathbf{w}$ , with 9 elements is formed from the first vector,  $\mathbf{a}$ , with the excluded fraction being the one for the *k*-th removed lamp. However, the sum of all fractions of this new vector,  $\mathbf{w}$ , is not equal to one. Therefore, the new vector,  $\mathbf{w}$ , is divided by the sum of its elements, such that its sum is now equal to one. Finally, the RMSE value is computed between the new vector,  $\mathbf{w}$ , and the second vector,  $\mathbf{x}$ .

## Results

Figure 1. shows the results of the unknown source analysis. The black dot on the plot denotes the mean of the RMSE values. The length of the vertical line, from the top bar to the bottom bar, is equal to two standard deviations of the RMSE values.



Figure 1. Results of the unknown source analysis. The horizontal axis shows which light source, present in the mixture, was removed in the spectral reference databank. The vertical axis shows the root mean-square error value of the method.

Clearly, when a light source is removed, the mean of the RMSE values is higher, when compared to the case when none of the light sources are removed. In other words, a light source present in the mixture spectrum, which is missing in the spectral reference database, degrades the results of the spectral unmixing method. The highest mean RMSE value of 12.7 % is reached when the compact fluorescent lamp is removed. The mean RMSE value of 8.1 % is reached when the white light-emitting diode, with a CCT of 2700 K, is removed. The lowest mean RMSE value of 4.4 % is reached when none of the light sources are removed.

To conclude:

- a) The compact fluorescent lamp (CFL) induces the largest error, when present in the mixture but is missing in the spectral reference database.
- b) To provide a measurement uncertainty of less than 10 % RMSE, all the light sources present in the mixture must be contained within the spectral reference database.